1   # Grade Four

2

3   In the years prior to grade four, students developed place value understandings,

4   generalized written methods for addition and subtraction, and added and subtracted

5   fluently within 1,000. They gained an understanding of single-digit multiplication and

6   division and became fluent with such operations. Students developed an understanding

7   of fractions as built up from unit fractions (Adapted from The Charles A. Dana Center

8   Mathematics Common Core Toolbox 2012).

9

10   ## WHAT STUDENTS LEARN IN GRADE FOUR

11   [Note: Sidebar]

Grade Four Critical Areas of Instruction

In grade four instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. (CCSSO 2010, Grade 4 Introduction).

Students also work toward fluency in addition and subtraction within 1,000,000 using the standard algorithm.

12

13   **Grade Four Standards for Mathematical Content**

14   The Standards for Mathematical Content emphasize key content, skills, and practices at

15   each grade level and support three major principles:

16   • **Focus:** Instruction is focused on grade level standards.

17   • **Coherence:** Instruction should be attentive to learning across grades and linking

18   major topics within grades.

19   • **Rigor:** Instruction should develop conceptual understanding, procedural skill and

20   fluency, and application.

21

22   Grade level examples of focus, coherence and rigor will be indicated throughout the

23   chapter.

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24

25 Not all of the content in a given grade is emphasized equally in the standards. Cluster

26 headings can be viewed as the most effective way to communicate the **focus** and

27 **coherence** of the standards. Some clusters of standards require a greater instructional

28 emphasis than the others based on the depth of the ideas, the time that they take to

29 master, and/or their importance to future mathematics or the later demands of college

30 and career readiness.

31

32 The following Grade 4 Cluster-Level Emphases chart highlights the content emphases

33 in the standards at the cluster level for this grade. The bulk of instructional time should

34 be given to "Major" clusters and the standards within them. However, standards in the

35 "Supporting" and "Additional" clusters should not be neglected. To do so will result in

36 gaps in students' learning, including skills and understandings they may need in later

37 grades. Instruction should reinforce topics in major clusters by utilizing topics in the

38 supporting and additional clusters. Instruction should include problems and activities

39 that support natural connections between clusters.

40

41 Teachers and administrators alike should note that the standards are not topics to be

42 checked off a list during isolated units of instruction, but rather content to be developed

43 throughout the school year through rich instructional experiences and presented in a

44 coherent manner (Adapted from the Partnership for Assessment of Readiness for

45 College and Careers [PARCC] 2012).

46

47 [**Note**: The Emphases chart should be a graphic inserted in the grade level section. The

48 explanation "key" needs to accompany it.]

49

50

51   **Grade 4 Cluster-Level Emphases**

52 **Operations and Algebraic Thinking**

53 • [m]: Use the four operations with whole numbers to solve problems. (**4.OA.1-3▲**)

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November 6, 2013. *The Mathematics Framework* has not been edited for publication.

54 • [a/s]: Gain familiarity with factors and multiples.[1] (**4.OA.4**)

55 • [a/s]: Generate and analyze patterns. (**4.OA.5**)

56
57 **Number and Operations in Base Ten**

58 • [m]: Generalize place value understanding for multi-digit whole numbers. (**4.NBT.1-3▲**)

59 • [m]: Use place value understanding and properties of operations to perform multi-digit
60   arithmetic. (**4.NBT.4-6▲**)

61
62 **Number and Operations—Fractions**

63 • [m]: Extend understanding of fraction equivalence and ordering. (**4.NF.1-2▲**)

64 • [m]: Build fractions from unit fractions by applying and extending previous understandings of
65   operations on whole numbers. (**4.NF.3-4▲**)

66 • [m]: Understand decimal notation for fractions, and compare decimal fractions. (**4.NF.5-7▲**)

67
68 **Measurement and Data**

69 • [a/s]: Solve problems involving measurement and conversion of measurements from a larger
70   unit to a smaller unit.[2] (**4.MD.1-2**)

71 • [a/s]: Represent and interpret data. (**4.MD.4**)

72 • [a/s]: Geometric measurement: understand concepts of angle and measure angles.
73   (**4.MD.5-7**)

74
75 **Geometry**

76 • [a/s]: Draw and identify lines and angles, and classify shapes by properties of their lines and
77   angles. (**4.G.1-3**)

78

Explanations of Major, Additional and Supporting Cluster-Level Emphases
Major[3] [m] clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.
Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade **Supporting [s]** clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis.   *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

---

[1] Supports students' work with multi-digit arithmetic as well as their work with fraction equivalence.
[2] Students use a line plot to display measurements in fractions of a unit and to solve problems involving addition and subtraction of fractions, connecting this work to the Number and Operations – Fractions clusters.
[3] The ▲symbol will indicate standards in a Major Cluster in the narrative.
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79 (Adapted from Smarter Balanced Assessment Consortia [Smarter Balanced], DRAFT

80 Content Specifications 2012)

81

82 **Connecting Mathematical Practices and Content**

83 The Standards for Mathematical Practice (MP) are developed throughout each grade

84 and, together with the content standards, prescribe that students experience

85 mathematics as a rigorous, coherent, useful, and logical subject that makes use of their

86 ability to make sense of mathematics. The MP standards represent a picture of what it

87 looks like for students to understand and do mathematics in the classroom and should

88 be integrated into every mathematics lesson for all students.

89

90 Although the description of the MP standards remains the same at all grades, the way

91 these standards look as students engage with and master new and more advanced

92 mathematical ideas does change. Below are some examples of how the MP standards

93 may be integrated into tasks appropriate for grade four students. (Refer to pages 9–13

94 in the "Overview of the Standards Chapters" for a complete description of the MP

95 standards.)

96

97   **Standards for Mathematical Practice (MP)**

98   **Explanations and Examples for Grade Four**

Standards for Mathematical Practice	Explanation and Examples
MP.1 Make sense of problems and persevere in solving them.	In grade four students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Students might use an equation strategy to solve the word problem. For example, students could solve the problem "Chris bought clothes for school. She bought 3 shirts for $12 each and a skirt for $15. How much money did Chris spend on her new school clothes?" with the equation $3 \times \$12 + \$15 = a$. Students may use visual models to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
MP.2 Reason abstractly and	Fourth graders recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They

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quantitatively.	extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts. Students might use array or area drawings to demonstrate and explain 154×6, as 154 added six times, and so develop an understanding of the distributive property. For example, $154 \times 6 = (100 + 50 + 4) \times 6 = (100 \times 6) + (50 \times 6) + (4 \times 6) = 600 + 300 + 24 = 924$. Teachers might ask, "How do you know" or "What is the relationship of the quantities?" to reinforce students' reasoning and understanding.
MP.3 Construct viable arguments and critique the reasoning of others.	Students may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?", "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations and ask questions. Students explain and defend their answers and solution strategies as they answer question that require an explanation.
MP.4 Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, and creating equations. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Students should be encouraged to answer questions, such as "What math drawing or diagram could you make and label to represent the problem?" or "What are some ways to represent the quantities?" Fourth graders evaluate their results in the context of the situation and reflect on whether the results make sense. For example, a student may use an area/array rectangle model to solve the following problem by extending from multiplication to division: A fourth grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?
MP.5 Use appropriate tools strategically.	Students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line or drawings of dimes and pennies to represent and compare decimals or protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units. Students should be encouraged to answer questions such as, "Why was it helpful to use…?"
MP.6 Attend to precision.	As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
MP.7 Look for and make use of structure.	Students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They generate number or shape patterns that follow a given rule. Teachers might ask, "What do you notice when…?" or "How do you know if something is a pattern?"

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MP.8 Look for and express regularity in repeated reasoning.	In grade four students notice repetitive actions in computation to make generalizations. Students use models to explain calculations and understand how algorithms work. Students examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions. Students should be encouraged to answer questions, such as "What is happening in this situation?" or "What predictions or generalizations can this pattern support?"

99   (Adapted from Arizona Department of Education [Arizona] 2012 and North Carolina

100   Department of Public Instruction [N. Carolina] 2011)

101

102   **Standards-based Learning at Grade Four**

103   The following narrative is organized by the domains in the Standards for Mathematical

104   Content and highlights some necessary foundational skills from previous grades and

105   provides exemplars to explain the content standards, highlight connections to the

106   various Standards for Mathematical Practice (**MP**), and demonstrate the importance of

107   developing conceptual understanding, procedural skill and fluency, and application. A

108   triangle symbol (▲) indicates standards in the major clusters (refer to the Grade 4

109   Cluster-Level Emphases chart on page #3).

110
111   **Domain: Operations and Algebraic Thinking**

112   Previously in grade three, students focused on concepts, skills, and problem solving

113   with single-digit multiplication and division (within 100). In grade four a critical area of

114   instruction is developing understanding and fluency with multi-digit multiplication and

115   developing understanding of division to find quotients involving multi-digit dividends.

116

Operations and Algebraic Thinking	4.OA

Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]
3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies

---

[1] See Glossary, Table 2.

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> including rounding.

117

118   In earlier grades students focused on addition and subtraction, and worked with additive

119   comparison problems (e.g., what *amount* would be added to one quantity in order to

120   result in the other: bigger quantity = smaller quantity + difference), in grade four

121   students compare quantities multiplicatively for the first time.

122

123   In a multiplicative comparison problem, the underlying structure is that a *factor*

124   multiplies one quantity to result in the other (e.g., b is n times as much as a,

125   represented by $n \times a = b$, bigger quantity = n × smaller quantity). Students interpret a

126   multiplication equation as a comparison and solve word problems involving

127   multiplicative comparison (**4.OA.1-2▲**) and should be able to identify and verbalize all

128   three quantities involved: which quantity is being multiplied (the smaller quantity), which

129   number tells how many times, and which number is the product (the bigger quantity).

130   Teachers should be aware that students often have difficulty with understanding the

131   order and meaning of numbers in multiplicative comparison problems, and so special

132   attention should be paid to understanding these types of problem situations (**MP.1**).

133

Example: Multiplicative Comparison Problems.
Unknown Product: "Sally is 5 years old. Her mother is 8 times as old as Sally is. How old is Sally's mother?" This problem takes the form $a \times b = ?$, where the factors are known but the product is unknown.
Unknown Factor (Group Size Unknown): "Sally's mother is 40 years old. That is 8 times as old as Sally is, How old is Sally?" This problem takes the form $a \times ? = p$, where the product is known, but the quantity being multiplied to become bigger, is unknown.
Unknown Factor 2 (Number of Groups Unknown): "Sally's mother is 40 years old. Sally is 5 years old. How many times older than Sally is this?" This problem takes the form $? \times b = p$, where the product is known but the multiplicative factor, which does the enlarging in this case, is unknown.

134

135   In grade four students solve three major common types of multiplication and division

136   problems, which are summarized in the following table.

137

138

	Unknown Product	Group Size Unknown (Partitive Division)	Number of Groups Unknown (Measurement Division)
	3 × 6 = ?	3 × ? = 18 and 18 ÷ 3 = ?	? × 6 = 18 and 18 ÷ 6 =?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement Example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, Area	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area Example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement Example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is three times as much as a blue hat costs. How much does a blue hat cost? *Measurement Example.* A rubber band is stretched to be 18 cm long and that is three times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement Example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	*a × ? = p*	*a × ? = p* and *p ÷ a = ?*	*? × b = p* and *p ÷ b = ?*

139 (CCSSI 2010, Glossary)[The table is also included in this Framework's Glossary.]

140

141 Students need many opportunities to solve contextual problems. A tape or bar diagram

142 can help students visualize and solve multiplication and division word problems. Tape

143 diagrams are useful for connecting what is happening in the problem with an equation

144 that represents the problem.

145

146

| Examples: Using Tape Diagrams to Represent Multiplication Compare Problems. |
|---|
| **Unknown Product:** "Skyler has 4 times as many books as Karen. If Karen has 36 books, how many books does Skyler have?"

Solution: If we represent the number of books that Karen has with a piece of tape, then the number of books Skyler has is represented by 4 pieces of tape of the same size. Students can represent this as $4 \times 36 = \square$.
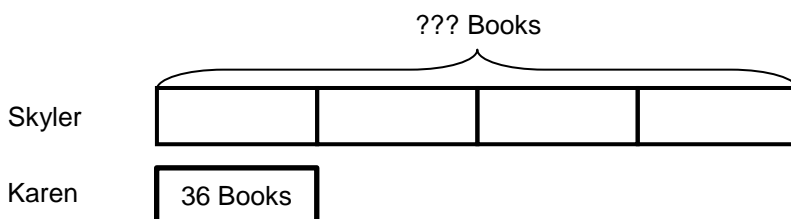
??? Books
Skyler
Karen   36 Books |
| **Unknown Factor (Group Size Unknown):** "Deborah sold 45 tickets to the school play, which is 3 times as many as Tomas sold. How many tickets did Tomas sell?"

Solution: Here, the number of books Deborah has (the product) is known and is represented by 3 pieces of tape. The number of tickets Tomas sold would be represented by one piece of tape. This representation helps students see that the equations $3 \times \square = 45$ or $45 \div 3 = \square$ represent the problem.
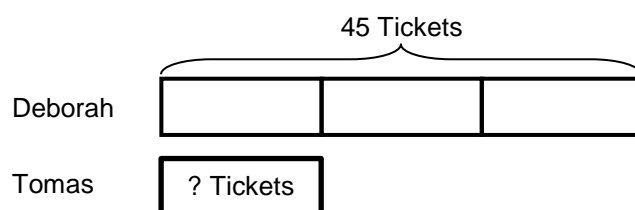
45 Tickets
Deborah
Tomas   ? Tickets |
| **Unknown Factor (Number of Groups Unknown):** "A used bicycle costs \$75 while a brand new one costs \$300. How many times as much does the new bike cost compared to the old bike?"

Solution: Here, the student represents the cost of the used bike by a piece of tape, and decides how many pieces of this tape will make up the cost of the new bicycle. The representation leads to the equations $\square \times 75 = 300$ and $300 \div 75 = \square$.

\$300
New   How many times as large?
Used   \$75 |
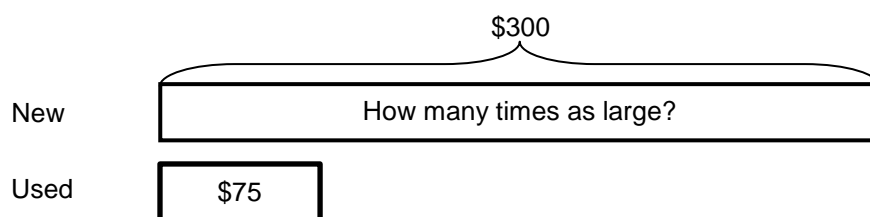
147

148 Additionally, students solve multi-step word problems using the four operations,

149 including problems in which remainders must be interpreted. (**4.OA.3▲**). Students use

150 estimation to solve problems. They identify when estimation is appropriate, determine

151 the level of accuracy needed to solve a problem and select the appropriate method of

152 estimation. This gives rounding usefulness, rather than making rounding a separate

153 topic that is covered arbitrarily.

154

Examples: Multi-step Word Problems and Strategies Called for in Standard 4.OA.3▲.
1. "There are 146 students going on a field trip. If each bus held 30 students, how many buses are needed?"   *Solution:* Since 150 ÷ 30 = 5, it seems like there should be around 5 buses. When we try to divide 146 by 30, we get 4 groups with 26 leftover. This means that $146 = 4 \times 30 + 26$. There are 4 filled with 30 students, with a fifth bus holding only 26 students. (In this case, one more than the quotient is the answer.)
2. "Suppose that 250 pencils were distributed equally among 33 students for a geometry project. What is the largest number of pencils each student can receive?"   *Solution:* Since 240 ÷ 30 = 8, it seems like each student should receive close to 8 pencils. When we divide 250 by 33, we get 7 with a remainder of 19. This means that $250 = 33 \times 7 + 19$. This tells us that each student can have 7 pencils with 19 leftover for the teacher to hold on to.
3. "Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each pack. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?"   *Solution:* "First, I multiplied 3 packs by 6 bottles per pack which equals 18 bottles. Then I multiplied 6 packs by 6 bottles per pack which is 36 bottles. I know 18 plus 36 is around 50. Since we're trying to get to 300, we'll need about 250 more bottles**."**

155

156   As students compute and interpret multi-step problems with remainders (**4.OA.3▲**),

157   they also reinforce important mathematical practices as they make sense of the problem

158   and reason about how the context connects to the four operations (**MP.1, MP.2**).

159

Common Misconceptions.
• Teachers may try to help their students by telling them that multiplying a number two numbers in a multiplicative comparison situation always makes the product *bigger*. While this is true with whole numbers greater than 1, it is *not true* when the first factor is a fraction smaller than 1 (or when the first factor is negative), something students will encounter in later grades. Teachers should be careful to emphasize that multiplying by a number *greater than 1* results in a product larger than the original number (**4.OA.1-2▲**).
• Students might be confused by the difference between six more than a number (additive) compared to six times a number (multiplicative). For example, using 18 and 6, a question could be "How much more is 18 than 6?" Thinking multiplicatively the answer is 3, however, thinking additively the answer is 12. (Adapted from KATM 4th FlipBook 2012).
• It is common practice when dividing numbers to write $250 \div 33 = 7$ R 19, for example. While this

notation has been used for quite some time, it obscures the relationship between the numbers in the problem, e.g., when students find fractional answers the correct equation becomes $250 \div 33 = 7\frac{19}{33}$. It is more accurate to write the answer in words, such as by saying, "when we divide 250 by 33, the quotient is 7 with 19 leftover," or to write the equation "$250 = 33 \times 7 + 19$," as in the example above. See standard **(4.NBT.6▲)**.

160

Operations and Algebraic Thinking   4.OA

Gain familiarity with factors and multiples.
4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

161

162 Students find all factor pairs for whole numbers in the range 1–100 **(4.OA.4).** They

163 extend the idea of decomposition to multiplication and learn to use the term *multiple*.

164 Any whole number is a multiple of each of its factors. For example, 21 is a multiple of 3

165 and a multiple of 7 because 21 = 3 × 7. A number can be multiplicatively decomposed

166 into equal groups and expressed as a product of these two factors (called factor pairs).

167 A *prime number* has only one and itself as factors. A *composite number* has two or

168 more factor pairs. The number 1 is neither prime nor composite. To find all factor pairs

169 for a given number, students need to search systematically, by checking if 2 is a factor,

170 then 3, then 4, and so on, until they start to see a "reversal" in the pairs (e.g., after

171 finding the pair 6 and 9 for 54, students will next find the reverse pair, 9 and 6). Knowing

172 how to determine factors and multiples is the foundation for finding common multiples

173 and factors in grade six (Adapted from The University of Arizona Progressions

174 Documents for the Common Core Math Standards [Progressions], K-5 CC and OA

175 2011).

176

Common Misconceptions.
- Students may think the number 1 is a prime number or that all prime numbers are odd numbers (counterexample: 2 has only 2 factors—1 and 2).
- When listing multiples of numbers students may not list the number itself. Students should be reminded that the smallest multiple is the number itself.
- Students may think larger numbers have more factors.

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Having students share all factor pairs and how they found them will help students avoid some of these misconceptions (Adapted from KATM 4th FlipBook 2012).

177

Focus, Coherence, Rigor:

The concepts and terms "prime" and "composite" are new at grade four. As students gain familiarity with factors and multiples (**4.OA.4**) they also reinforce and support major work at the grade, such as multi-digit arithmetic in the cluster "Use place value understanding and properties of operations to perform multi-digit arithmetic" (**4.NBT.4-6▲**) and fraction equivalence in the cluster "Extend understanding of fraction equivalence and ordering" (**4.NF.1-2▲**).

178

Operations and Algebraic Thinking   4.OA

Generate and analyze patterns.
  5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

179

180   Understanding patterns is fundamental to algebraic thinking. In grade four students

181   generate and analyze number and shape patterns that follow a given rule **(4.OA.5)**.

182   Students begin by reasoning about patterns, connecting a rule for a given pattern with

183   its sequence of numbers or shapes. A pattern is a sequence that repeats or evolves in a

184   predictable process over and over. A rule dictates what that process will look like.

185   Patterns that consist of repeated sequences of shapes or growing sequences of

186   designs can be appropriate for the grade.

187

188   For example, students could examine a sequence of dot designs in which each design

189   has 4 more dots than the previous one and then reason about how the dots are

190   organized in the design to determine the total number of dots in the 100th design.

191   **(MP.2, MP.4, MP.5, MP.7)** (Adapted from Progressions K-5 CC and OA 2011).

192

193   Following are examples of problems that can help students understand patterns:

194   "Double Plus One" available at http://illustrativemathematics.org/standards/k8#

195   (Illustrative Mathematics 2013).

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196

197   "Patterns that Grow" available

198   at http://illuminations.nctm.org/LessonDetail.aspx?ID=U103 (National Council of

199   Teachers of Mathematics [NCTM] Illuminations 2013).

200

Focus, Coherence, Rigor:

Numerical patterns (**4.OA.5**) allow students to reinforce facts and develop fluency with operations and support major work at the grade in the cluster "Use place value understanding and properties of operations to perform multi-digit arithmetic" (**4.NBT.4-6▲**).

201
202   **Domain: Number and Operations in Base Ten**
203
204   In grade four, students extend their work in the base-ten number system and generalize

205   previous place value understanding to multi-digit whole numbers (less than or equal to

206   1,000,000).

207

Numbers and Operations in Base Ten	4.NBT

Generalize place value understanding for multi-digit whole numbers.
1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

208

209   Students read, write, and compare numbers based on the meaning of the digits in each

210   place (**4.NBT.1-2▲**). In the base-ten system, the value of each place is 10 times the

211   value of the place to the immediate right. By reasoning that each unit in a place

212   becomes one unit in the next left place (because it is multiplied by ten), students can

213   come to see and understand that multiplying by 10 yields a product in which each digit

214   of the multiplicand is shifted one place to the left (Adapted from Progressions K-5 NBT

215   2011).

216

217   Students need multiple opportunities to use real-world contexts to read and write multi-

218   digit whole numbers. Student need to reason about the magnitude of digits in a number

219   and analyze the relationships of number. They can build larger numbers by using graph

220   paper with very small squares and labeling examples of each place with digits and

221   words (e.g., ten thousand and 10,000).

222

223   To read and write numerals between 1,000 and 1,000,000, students need to understand

224   the role of commas. Each sequence of three digits made by commas is read as

225   hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit

226   (e.g., thousand, million). Layered place value cards such as those used in earlier

227   grades can be put on a frame with the base-thousand units labeled below. Then cards

228   forming hundreds, tens, and ones can be placed on each section and the name read off

229   using the card values followed by the word "million", then "thousand", then the silent

230   ones **(MP.2, MP.3, MP.8).**

231

232   Fourth-grade students build on the grade-three skill of rounding to the nearest 10 or 100

233   to round multi-digit numbers and to make reasonable estimates of numerical values.

234   (**4.NBT.3▲**).

235

Example: Rounding Numbers in Context. (MP.4)
The population of Midtown, U.S.A., was last recorded to be 76,398. The city council wants to round the population to the nearest thousand for a business brochure. What number should they round the population to?
Solution: When students represent numbers stacked vertically, they can see the relationships between the numbers more clearly. Students might think: "I know the answer is either 76,000 or 77,000. If I write 76,000 below 76,398 and 77,000 above it, I can see that the midpoint is 76,500, which is *above* 76,398. This tells me they should round the population to 76,000."

77,000
76,398
76,000

236

<table>
<tr><td style="background:blue;color:white">**Numbers and Operations in Base Ten**</td><td style="background:blue;color:white">**4.NBT**</td></tr>
</table>

Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.   Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5.   Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-

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> digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
> 6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

237

238 At grade four, students become fluent with addition and subtraction with multi-digit

239 whole numbers to 1,000,000 using standard algorithms (**4.NBT.4▲**). A central theme in

240 multi-digit arithmetic is to encourage students to develop methods they understand, can

241 explain, and can think about, rather than merely following a sequence of directions,

242 rules or procedures they do not understand. In previous grades, students built a

243 conceptual understanding of addition and subtraction with whole numbers as they

244 applied multiple methods to compute and solve problems. The emphasis in grade four is

245 on the power of the regular one-for-ten trades between adjacent places that let students

246 extend a method they already know to many places. Because students in grades two

247 and three have been using at least one method that will generalize to 1,000,000, this

248 extension in grade four should not have to take a long time. Thus, students will also

249 have sufficient time for the major new topics of multiplication and division (**4.NBT.5-**

250 **6▲**).

251   [Note: Sidebar]

Fluency
In kindergarten through grade six there are individual content standards that set expectations for fluency with computations using the standard algorithm (e.g., "fluently" add and subtract multi-digit whole numbers using the standard algorithm (**4.NBT.4▲**)). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding (such as reasoning about quantities, the base-ten system, and properties of operations), thoughtful practice, and extra support where necessary.
The word "fluent" is used in the standards to mean "reasonably fast and accurate" and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies (Adapted from Progressions K-5 CC and OA 2011 and PARCC 2012).

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252

253   In grade four students extend multiplication and division to include whole numbers

254   greater than 100. Students should use methods they understand and can explain to

255   multiply and divide. The standards (**4.NBT.5-6▲**) call for students to use visual

256   representations such as area and array models that students draw and connect to

257   equations and written numerical work that supports student reasoning and explanation

258   of methods. By reasoning repeatedly about the connections between math drawings

259   and written numerical work, students can come to see multiplication and division

260   algorithms as abbreviations or summaries of their reasoning about quantities.

261

262   After students have discussed how to show an equal groups situation or a multiplication

263   compare situation with an area model, they can use area models for any multiplication

264   situation. The rows represent the equal groups of objects or the larger compared

265   quantity and students imagine that the objects in the situation lie in the squares and so

266   form an array. Such array models become too difficult to draw, so students can make

267   sketches of rectangles and then label the resulting product as the number of things or

268   square units. When using area models to represent an actual area situation, the two

269   factors are in length units (e.g., cm) while the product is in square units (e.g., cm^2).

270

271

Example: Area Models and Strategies for Multi-digit Multiplication , Single Digit Multiplier (4.NBT.5▲)

"Chairs are being set up for a small play. There should be 3 rows of chairs and 14 chairs in each row. How many chairs will be needed?"

Solution: As in grade three, when students first made the connection between array models and the area model, students might start by drawing a sketch of the situation. They can then be reminded to see the chairs as if surrounded by unit squares and hence a model of a rectangular region. With base-ten blocks or math drawings **(MP.2, MP.5)**, students abstract the problem and see it being broken down into $3 \times (10 + 4)$.
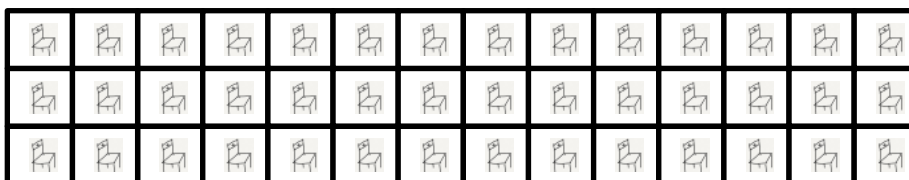
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Making a sketch like the one above becomes cumbersome, so students move toward representing such drawings abstractly, with rectangles, as shown to the right. This builds on the work begun in grade 3. Such diagrams help children see the distributive property: "3×14 can be written as $3 \times (10 + 4)$, and I can do the multiplications separately and add the results, $3 \times (10 + 4) = 3 \times 10 + 3 \times 4$. The answer is $30 + 12 = 42$, or 42 chairs."

272
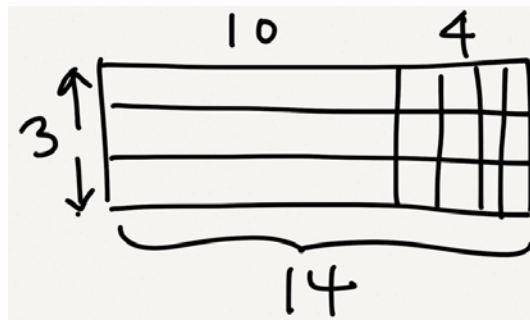
273 In grade three students worked with multiplying single digit numbers by multiples of 10

274 **(3.NBT.3)**. This idea is extended in grade four, e.g., since $6 \times 7 = 42$, it must be true

275 that:

276 - $6 \times 70 = 420$, since this is "six times seven tens," which is 42 tens,

277 - $6 \times 700 = 4200$, since this is "six times seven hundreds," which is 42 hundreds,

278 - $6 \times 7000 = 42,000$, since this is "six times seven thousands," which is 42

279   thousands,

280 - $60 \times 70 = 4200$, since this is "sixty times seven tens," which is 420 tens, or 4200.

281 Math drawings and base-ten blocks support the development of these *extended*

282 *multiplication facts*. The ability to find products such as these is important when using

283 variations of the standard algorithm for multi-digit multiplication, described below.

284

Examples: Developing Written Methods for Multi-Digit Multiplication. (4.NBT.5▲)

| Left to right | Right to left | Right to left |

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Find the
6×729.

		showing the partial products		showing the partial products		recording the "carries" below		product:

Find the 6×729.

```
      729
   ×    6    t□inking:
   4200   6 × 7 hundreds
    120   6 × 2□tens
     54   6 × 9
   4374
```

```
      729
    × 6
     54   6 × 9
    120   6 × 2 tens
   4200   6 × 7 hundreds
   4374
```

```
      729
    × 6
      1 5
   4224
   4374
```

Sufficient practice with drawing rectangles (or constructing them with base-ten blocks) will help students understand

729 =	700	+ 20	+ 9

6 | $6 \times 700 =$ 6 groups of 7 hundreds = 42 hundreds = 4200 | $6 \times 20 =$ 6 groups of 2 tens = 12 tens = 120 | $6 \times 9 = 54$ |
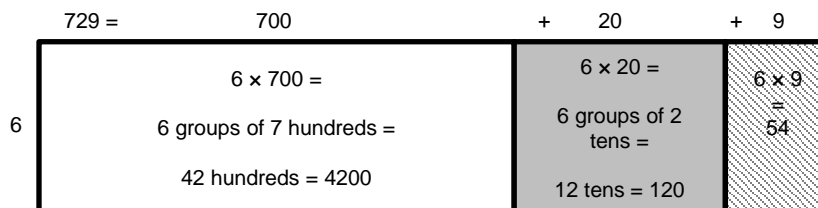
that the problem can be represented with a rectangle such as the one shown. The product is given by the total area: $6 \times 729 = 6 \times 700 + 6 \times 20 + 6 \times 9$. Understanding extended multiplication facts allows students to find the *partial products* quickly. Student can record the multiplication in several ways:

Find the product: 27×65.

Solution: This time, a rectangle is drawn, and "like" base-ten units (e.g., tens and ones) are represented by sub-regions of the rectangle. Repeated use of the distributive property shows that:

$$27 \times 65 = (20 + 7) \times 65 = 20 \times 65 + 7 \times 65$$
$$= 20 \times (60 + 5) + 7 \times (60 + 5)$$
$$= 20 \times 60 + 20 \times 5 + 7 \times 60 + 7 \times 5.$$

	60	+ 5

20 | $20 \times 60 =$ 2 tens times 6 tens= 12 hundreds = 1200 | $20 \times 5 =$ 2 tens x 5= 10 tens= 100 |
+ | | |
7 | $7 \times 60 =$ 7 x 6 tens = 42 tens = 420 | $7 \times 5 = 35$ |
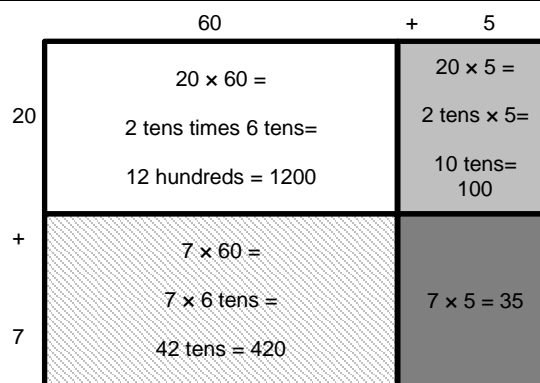
The product is again given by the total area:
$$1200 + 100 + 420 + 35 = 1755.$$
Below are two written methods for recording the steps of the multiplication.

Showing the partial products

```
      65
   × 27    thinking:
      35   7 × 5
     420   7 × 6 tens
     100   2 tens × 5
    1200   2 tens × 6 tens
    1755
```

Recording the carries below for correct place value placement

```
       65
     × 27
      4 3
      35
     1 1
     200
    1755
```

Notice that the boldfaced **0** is included in the second method, indicating that we are multiplying not just by 2 in this row, but by 2 *tens.*

285

286 General methods for computing quotients of multi-digit numbers and one-digit numbers

287 **(4.NBT.6▲)** rely on the same understandings as for multiplication, but these are cast in

288 terms division. For example, students may see division problems as knowing the area of

289 a rectangle but not one side length (the quotient) or as finding the size of a group when

290 the number of groups is known (measurement division).

291

292

293

Example: Using the Area Model to Develop Division Strategies.

Find the quotient: 750 ÷ 6.

Solution: "Just like with multiplication, I can set this up as a rectangle, but with one side unknown since this is the same as $?? \times 6 = 750$. I find out what the number of hundreds would be for the unknown side length; that's 1 hundred or 100, since 100 × 6 = 600 and that's as large as I can go. Then, I have $750 - 600 = 150$ square units left, so I find the number of tens that are in the other side. That's 2 tens or 20, since 20 × 6 = 120. Last, there are $150 - 120 = 30$ square units left, so the number of ones on the other side must be 5 since $5 \times 6 = 30$."

One way students can record this is shown, wherein *partial quotients* are stacked atop one another, with 0s included to indicate place value and as a reminder of how students obtained the numbers. The full quotient is the sum of these stacked numbers.

? hundreds + ? tens + ? ones

6

100   +   20   + 5

6
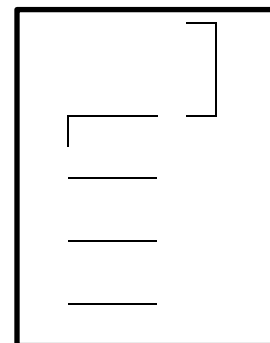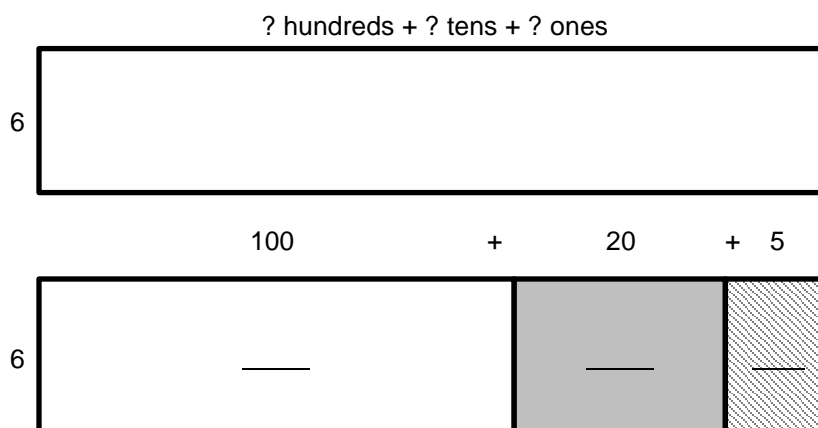
294

295   General methods for multi-digit division computation include decomposing the dividend

296   into like base-ten units and finding the quotient unit by unit, starting with the largest unit

297   and continuing on to smaller units. As with multiplication, this relies on the distributive

298   property. This work will continue in grade five and culminate in fluency with the standard

299   algorithm in grade six (Adapted from PARCC 2012).

300

301   In grade four students also find whole number quotients with remainders **(4.NBT.6▲).**

302   When students experience finding remainders, they should learn the appropriate way to

303   write the result. For instance, students divide and find that $195 \div 9 = 21$ with 6 leftover.

304   This can be written as $195 = 21(9) + 6$. When put into a context, the latter equation

305   makes sense. For instance, if 195 books are distributed equally among 9 classrooms,

306   then each classroom gets 21 books with 6 books leftover. The equation $195 = 21(9) +$

307   6 is closely related to the equation $195 \div 9 = 21\frac{6}{9}$ which students will write in later

308   grades. The notation $195 \div 9 = 21$ R 6 is best avoided.

309

310   As students decompose numbers to solve multiplication problems they also reinforce

311   important mathematical practices such as seeing and making use of structure (**MP.7**).

312   As they illustrate and explain calculations they model (**MP.4**), use appropriate drawings

313   as tools strategically (**MP.5**) and attend to precision (**MP.6**) using base-ten units.

314

315   Following is a sample problem that connects the Standards for Mathematical Content

316   and the Standards for Mathematical Practice.

Standards	Explanations and Examples
4.NBT.5: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and properties of operations. Illustrate and explain the calculation using equations, rectangular arrays, and/or area models. 4.MD.3: Apply the area and perimeter formals for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*	**Sample Problem:** What are the areas of the four sections of Mr. Griffin's backyard? There is a grass lawn, a flower garden, a tomato garden, and a stone patio. What is the area of his entire backyard? How did you find your answer? **Solution:** The areas of the four sections are 100 sq. ft., 80 sq. ft., 40 sq. ft., and 32 sq. ft. respectively. The area of the entire backyard is the sum of these areas, (100+80+40+32) sq. ft., or 252 sq. ft. This is the same as finding the product (18×14) sq. ft. **Classroom Connections:** The purpose of this task is to illuminate the connection between the area of a rectangle as representing the product of two numbers and the partial products algorithm for multiplying multi-digit numbers. In this algorithm, which is shown to the right, each digit of one number is multiplied by the each digit of the other number and the "partial products" are written down. The sum of these partial products is the product of the original numbers. Place value can be emphasized by specifically reminding students that if we multiply the two 10s together, since each represents one 10, their product is 100. Finally, the area model provides a visual justification for how the algorithm works. **Connecting to the Standards for Mathematical Practice:** (MP.1) Students make sense of the problem when they see that the measurements on the side and top of the diagram persist and yield the measurements of the smaller areas. (MP.2) Students reason abstractly as they represent the areas of the yard as multiplication problems to be solved. (MP.5) Students use appropriate tools strategically when they apply the formula for the area of a rectangle to solve the problem. They organize their work in a way that makes sense to them. (MP.7) Teachers can use this problem and similar problems to illustrate the distributive property of multiplication. In this case, we have that 18×14 = (10×14) + (8×14) = (10×10) + (10×4) + (8×10) + (8×4).

18
× 14
Area of Stone Patio→ 32 (4×8)
Area of Tomato Garden→ 40 (4×10)
Area of Flower Garden→ 80 (10×8)
Area of Grass Lawn→100 (10×10)
Area of Entire Backyard→ 252 (14×18)

317

318

319   **Domain: Number and Operations—Fractions[4]**

320

321   Student proficiency with fractions is essential to success in algebra at later

322   grades. In grade three students developed an understanding of fractions as built

323   from unit fractions. A critical area of instruction in grade four is developing an

324   understanding of fraction equivalence, addition and subtraction of fractions with

325   like denominators, and multiplication of fractions by whole numbers.

326

Numbers and Operations—Fractions	4.NF
Extend understanding of fraction equivalence and ordering. 1. Explain why a fraction *a/b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	

327

328   Grade four students learn a fundamental property of equivalent fractions:

329   multiplying the numerator and denominator of a fraction by the same non-zero

330   whole number results in a fraction that represents the same number as the

331   original fraction (e.g., $\frac{a}{b} = \frac{n \times a}{n \times b}$, for $n \neq 0$). Students use visual fraction models,

332   with attention to how the number and size of the parts differ even though the two

333   fractions themselves are the same size (**4.NF.1▲**). This property forms the basis

334   for much of the work with fractions in fourth grade; including comparing, adding,

335   and subtracting fractions and the introduction of finite decimals.

336

337   Students reason about and explain why fractions are equivalent using visual

338   models. For example, the area models below all show fractions equivalent to $\frac{1}{2}$,

339   and while in grade three students simply justified that all the models represent

340   the same amount visually, in grade four students reason about *why* it is true that

---

341 $\frac{1}{2} = \frac{2\times1}{2\times2} = \frac{3\times1}{3\times2} = \frac{4\times1}{4\times2}$, etc. They use reasoning such as: when a horizontal line is

342 drawn through the center of the first model to obtain the second, both the number

343 of equal parts and the number of those parts we are counting double ($2 \times 2 = 4$

344 in the denominator, $2 \times 1 = 2$ in the numerator, respectively), but even though

345 there are more parts counted they are smaller parts. Students notice

346 connections between the models and the fractions they represent in the way both

347 the parts and wholes are counted and begin to generate a rule for writing

348 equivalent fractions. Students also emphasize the inversely related changes: the

349 number of unit fractions becomes larger, but the size of the unit fraction becomes

350 smaller.

$$\frac{1}{2}$$

$$\frac{2}{4} = \frac{2 \times 1}{2 \times 2}$$

$$\frac{3}{6} = \frac{3 \times 1}{3 \times 2}$$

$$\frac{4}{8} = \frac{4 \times 1}{4 \times 2}$$
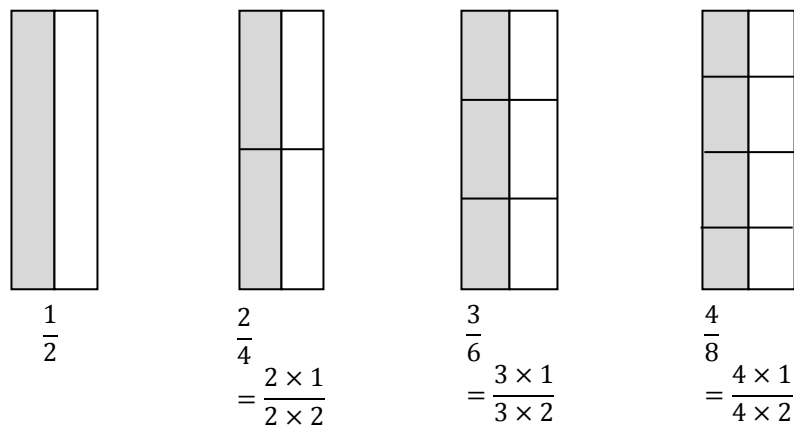
351

352 (Adapted from Arizona 2012)

353

354 Students should have repeated opportunities to use pictures such as these and

355 the ones below to understand the general method for finding equivalent fractions.

356 Of course, students may also come to see that the rule works both ways, for

357 example:

$$\frac{28}{35} = \frac{7 \times 4}{7 \times 5} = \frac{4}{5}.$$

358 Teachers must be careful to not overemphasize this "simplifying" of fractions, as

359 there is no mathematical reason for doing so, though depending on the problem

360 context one form may be more desirable. In particular, teachers should avoid the

361 use of the term "reducing" fractions for this process, as the value of the fraction

362 itself is *not* being reduced. A more neutral term such as "renaming" (which hints

363   to these fractions simply being different names for the same amount) allows for

364   referring to this strategy without the potential for student misunderstanding.

365

366   [Note: Sidebar]


Focus, Coherence, Rigor:

While it is true that one can justify that $\frac{a}{b} = \frac{n \times a}{n \times b}$ by arguing that:

$$\frac{n \times a}{n \times b} = \frac{n}{n} \times \frac{a}{b} = 1 \times \frac{a}{b} = \frac{a}{b}$$

i.e., that we are simply multiplying by 1 in the form of $\frac{n}{n}$, since students *have not yet encountered* the general notion of fraction multiplication in fourth grade, this argument should be avoided in favor of developing an understanding with diagrams and reasoning about the size and number of parts that are created in this process. Students will learn the general rule that $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ in grade five.

367

Examples: Reasoning With Diagrams That $\frac{a}{b} = \frac{n \times a}{n \times b}$.

Using an Area Model: The whole is the rectangle, measured by its area. The picture on the left shows the area divided into three rectangles of equal area (thirds) with two of them shaded (2 pieces of size $\frac{1}{3}$), representing $\frac{2}{3}$. On the right, the vertical lines divide the parts (the thirds) into smaller parts. There are now 4×3 smaller rectangles of equal area, and the shaded area now comprises 4×2 of them, so it represents $\frac{4 \times 2}{4 \times 3}$.
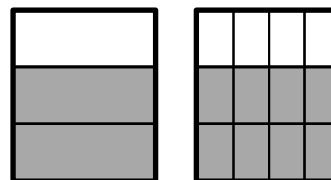
Using a Number Line: The top number line shows $\frac{4}{3}$: it is 4 parts when the unit length is divided into three equal parts and then iterated. When each of the intervals of length $\frac{1}{3}$ is further divided into 5 equal parts, there are now 5×3 of these new equal parts in the unit interval. Since 4 of the $\frac{1}{3}$ parts were circled before, and each of these has been subdivided into 5 parts, there are now 5×4 of these new small parts. Therefore $\frac{4}{3} = \frac{5 \times 4}{5 \times 3} = \frac{20}{15}$.

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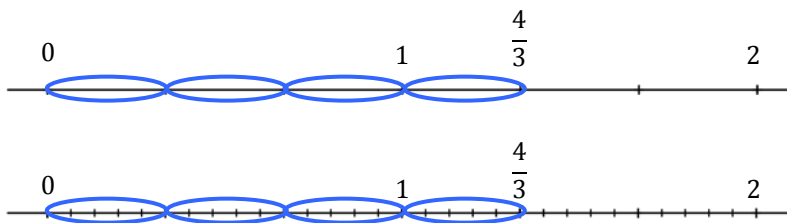
368   (Above examples adapted from Progressions 3-5 NF 2012)

369

370   Creating equivalent fractions by dividing and shading squares or circles, and

371   matching each fraction to its location on the number line can reinforce students'

372   understanding of fractions. For example, see "Equivalent Fractions" available

373   at http://illuminations.nctm.org/activitydetail.aspx?id=80 (NCTM Illuminations

374   2013).

375   Students apply their new understanding of equivalent fractions to compare two

376   fractions with different numerators and different denominators (**4.NF.2▲**). They

377   compare fractions using benchmark fractions, and by finding common

378   denominators or common numerators. Students explain their reasoning and

379   record their results using >, < and = symbols.

380

Examples: Comparing Fractions.
1. Students might compare fractions to benchmark fractions, e.g. comparing to $\frac{1}{2}$ when comparing $\frac{3}{8}$ and $\frac{2}{3}$. Students see that $\frac{3}{8} < \frac{4}{8} = \frac{1}{2}$, and that since $\frac{2}{3} = \frac{4}{6}$ and $\frac{4}{6} > \frac{3}{6} = \frac{1}{2}$, it must be true that $\frac{3}{8} < \frac{2}{3}$.
2. Students compare $\frac{5}{8}$ and $\frac{7}{12}$ by writing them with a common denominator. They find that $\frac{5}{8} = \frac{5 \times 12}{8 \times 12} = \frac{60}{96}$ and $\frac{7}{12} = \frac{7 \times 8}{12 \times 8} = \frac{56}{96}$ and reason therefore that $\frac{5}{8} > \frac{7}{12}$. Notice that students do not need to find the smallest common denominator for two fractions; any one will work.
3. Students can also find a common numerator to compare $\frac{5}{8}$ and $\frac{7}{12}$. They find that $\frac{5}{8} = \frac{5 \times 7}{8 \times 7} = \frac{35}{56}$ and $\frac{7}{12} = \frac{7 \times 5}{12 \times 5} = \frac{35}{60}$. They then reason that since parts of size $\frac{1}{56}$ are larger than parts of size $\frac{1}{60}$ when the whole is the same, that $\frac{5}{8} > \frac{7}{12}$.

381

382

Numbers and Operations—Fractions	**4.NF**

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
3.   Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*.
    a.   Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
    b.   Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8*

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> *+ 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
> c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
> d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

383

384 In grade four students extend previous understanding of addition and subtraction

385 of whole numbers to add and subtract fractions with like denominators

386 **(4.NF.3▲)**. They begin by understanding a fraction $\frac{a}{b}$ as a sum of the unit

387 fractions $\frac{1}{b}$. In grade three, students learned that the fraction $\frac{a}{b}$ represented a

388 parts when a whole is broken into b equal parts (i.e., parts of size $\frac{1}{b}$.) However, in

389 grade four, students connect this understanding of a fraction with the operation of

390 addition; for instance, they see now that if a whole is broken into 4 equal parts

391 and 5 of them are taken, then this is represented by both $\frac{5}{4}$ and the expression

392 $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ **(4.NF.3b▲)**. They experience composing fractions from and

393 decomposing fractions into sums of unit fractions and non-unit fractions in this

394 general way, e.g., by seeing $\frac{5}{4}$ also as

395   • $\frac{1}{4} + \frac{1}{4} + \frac{3}{4}$

396   • $\frac{2}{4} + \frac{3}{4}$

397   • $\frac{1}{4} + \frac{3}{4} + \frac{1}{4}$, etc.

398 Working with this standard supports student learning of **(4.NF.3a▲)** and

399 **(4.NF.3d▲)** by writing and using unit fractions. It also helps students avoid the

400 common misconception of adding two fractions by adding their numerators and

401 denominators, e.g. erroneously writing $\frac{1}{2} + \frac{5}{6} = \frac{6}{8}$. Work with **(4.NF.3b▲)** helps

402 students see that the unit fraction for the total is the same as the unit fractions
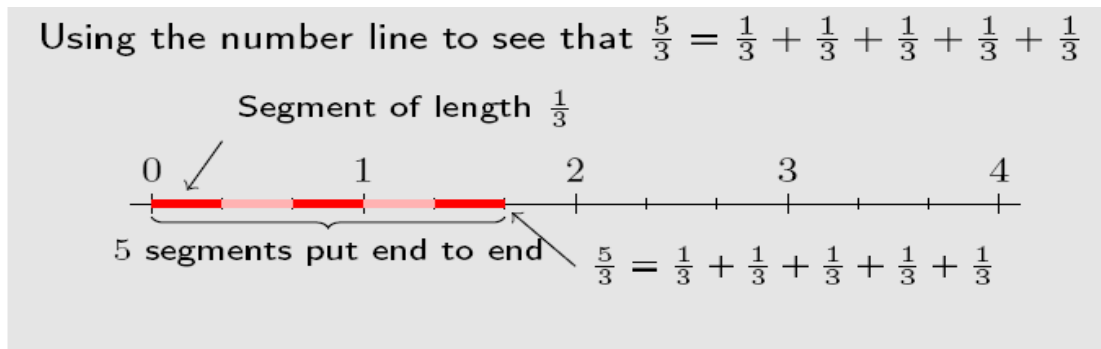
403 being added and grouped into fractions made from that unit fraction. In general,

404 the meaning of addition is the same for both fractions and whole numbers.

405 Students understand addition as "putting together" like units and they visualize

406   how fractions are built from unit fractions and that a fraction is a sum of unit

407   fractions.

408

409   Students may use visual models to support this understanding, for example,

410   showing that $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ by using a number line model. **(MP.1, MP.2,**

411   **MP.4, MP.6, MP.7).**

Using the number line to see that $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

Segment of length $\frac{1}{3}$

0   1   2   3   4

5 segments put end to end $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

412

413   (Source: Progressions 3-5 NF 2012)

414

415   Students add or subtract fractions with like denominators, including mixed

416   numbers **(4.NF.3a, c▲)** and solve word problems involving fractions **(4.NF.3d▲).**

417   They connect their understanding of any fraction as being composed of unit

418   fractions to realize that, for example:

$$\frac{7}{5} + \frac{4}{5} = \overbrace{\frac{1}{5} + \cdots + \frac{1}{5}}^{7} + \overbrace{\frac{1}{5} + \cdots + \frac{1}{5}}^{4} = \overbrace{\frac{1}{5} + \cdots + \frac{1}{5}}^{7+4} = \frac{7+4}{5}.$$

419   This quickly allows students to develop a general principle that $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$.

420   Using similar reasoning, students understand that $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$.

421

422   Students also compute sums of whole numbers and fractions, by realizing that

423   any whole number can be written as an equivalent number of unit fractions of a

424   given size, e.g. they find the sum $3 + \frac{7}{2}$ in the following way:

$$3 + \frac{7}{2} = \frac{6}{2} + \frac{7}{2} = \frac{13}{2}.$$

425   Understanding this method of adding a whole number and fraction allows

426   students to accurately convert mixed numbers into fractions, e.g.:

$$4\frac{5}{8} = 4 + \frac{5}{8} = \frac{32}{8} + \frac{5}{8} = \frac{37}{8}.$$

427   Students should develop a firm understanding that a mixed number indicates the

428   sum of a whole number and a fraction (i.e., $a\frac{b}{c} = a + \frac{b}{c}$), and should learn a

429   method for converting them to fractions that is connected to the meaning of

430   fractions such as the one above, rather than typical rote methods.

Examples: Reasoning With Addition and Subtraction of Fractions. (4.NF.3a-d▲).

1. Mary and Lacey share a pizza. Mary ate $\frac{3}{6}$ of the pizza and Lacey ate

    $\frac{2}{6}$ of the pizza. How much of the pizza did the girls eat altogether?

    Use the picture of a pizza to explain your answer.

    Solution: "I labeled three sixths for Mary and two sixths for Lacey. I

    can see that altogether they've eaten $\frac{5}{6}$ of the pizza. Also, I know that

    $$\frac{3}{6} + \frac{2}{6} = \frac{2+3}{6} = \frac{5}{6}."$$
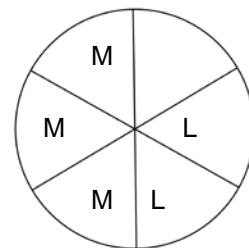
2. Susan and Maria need $8\frac{3}{8}$ feet of ribbon to package gift baskets. Susan has $3\frac{1}{8}$ feet of ribbon

    and Maria has $5\frac{3}{8}$ feet of ribbon. How much ribbon do they have altogether? Is it enough to

    complete the packaging?

    Solution: "I know I need to find $5\frac{3}{8} + 3\frac{1}{8}$ to find out how much they have altogether. I know

    that altogether they have $3 + 5 = 8$ feet of ribbon plus the other $\frac{1}{8} + \frac{3}{8}$ feet of ribbon.

    Altogether this is $8\frac{4}{8}$ feet of ribbon, which means they have enough ribbon to do their

    packaging. They even have $\frac{1}{8}$ feet of ribbon left."
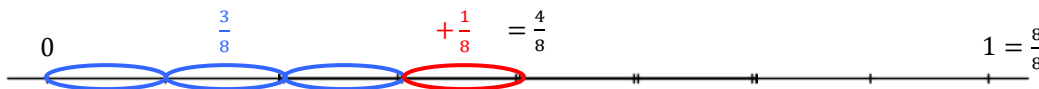
3. Elena, Matthew, and Kevin painted a wall. Elena painted $\frac{5}{9}$ of the wall and Matthew painted $\frac{3}{9}$

    of the wall. Kevin paints the rest. How much of the

    wall does Kevin paint? Use the picture to help find

    your answer.

    Solution: "I can show in the picture that Elena and

    Matthew painted $\frac{8}{9}$ altogether by shading what Elena

---
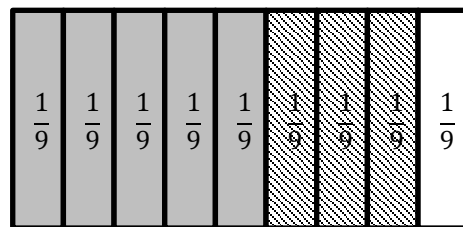
and Matthew painted. The remaining that Kevin paints is $\frac{1}{9}$. I can write this as $1 - \frac{8}{9} = \frac{1}{9}$, or

even $1 - \frac{5}{9} - \frac{3}{9} = \frac{1}{9}$." (New York State Education Department [NYSED] 2012).

431

Numbers and Operations—Fractions	4.NF

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
   b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

432

433 Previously in grade three, students learned that 3 × 7 can be represented as the

434 total number of objects in 3 groups of 7 objects, and that they could find this by

435 finding the sum 7 + 7 + 7. Grade four students apply this concept to fractions,

436 understanding a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ (**4.NF.4a▲**). Intimately connected

437 with standard **(4.NF.3)**, students make the shift to seeing $\frac{5}{3}$ as $5 \times \frac{1}{3}$, for example

438 by seeing:

$$\frac{5}{3} = \overbrace{\frac{1}{3} + \cdots + \frac{1}{3}}^{5 \text{ times}} = 5 \times \frac{1}{3}$$

439 Students then extend this understanding to make

440 meaning of the product of a whole number and a

441 fraction (**4.NF.4b▲**), for example, by seeing $3 \times \frac{2}{5}$

442 as:

$$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{3 \times 2}{5} = \frac{6}{5}.$$

443   (Progressions 3-5 NF 2012)

444
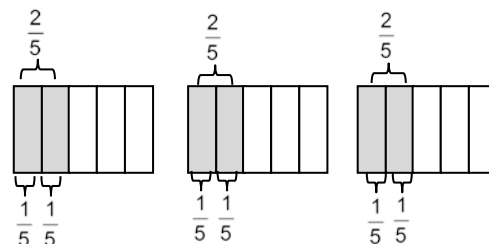
445   Students are presented with opportunities to work with problems involving

446   multiplication of a fraction by a whole number in context to relate situations,

447   models, and corresponding equations (**4.NF.4c▲**).

448

Example: Multiplying a Fraction by a Whole Number (4.NF.4c▲).

Each person at a dinner party eats $\frac{3}{8}$ of a pound of pasta. There are 5 people at the party. How many pounds of pasta are needed? Pasta comes in 1-lb boxes. How many boxes should be bought?
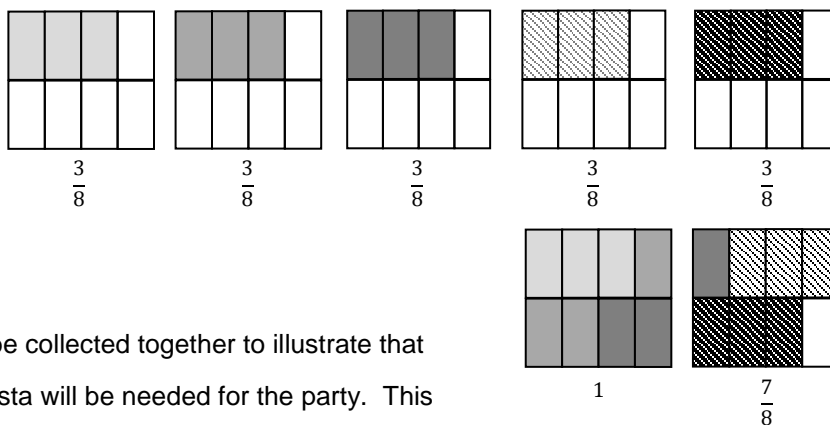
Solution: If five rectangles are drawn, with $\frac{3}{8}$ of a pound shaded in each rectangle, then students see that they are finding $5 \times \frac{3}{8} = \frac{15}{8}$.

The separate eighths can be collected together to illustrate that altogether $1\frac{7}{8}$ pounds of pasta will be needed for the party. This means that 2 boxes should be bought.

---

449   (Adapted from Arizona 2012 and N. Carolina 2011)

450

Numbers and Operations—Fractions   4.NF

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4]
    For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.

6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using **the number line or another** visual model. **CA**

---

[4] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

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451

452   In fourth grade students develop an understanding of decimal notation for

453   fractions and compare *decimal fractions* (fractions with denominator 10 or 100).

454   This work lays the foundation for performing operations with decimal numbers in

455   grade five. Students learn to add decimal fractions by converting them to

456   fractions with the same denominator (**4.NF.5▲**). For example, students express

457     $\frac{3}{10}$ as $\frac{30}{100}$ before they add $\frac{30}{100} + \frac{4}{100} = \frac{34}{100}$. Students can use base ten blocks,

458   graph paper, and other place value models to explore the relationship between

459   fractions with denominators of 10 and 100 (Adapted from Progressions 3-5 NF

460   2012).

461

462   In grade four, students first use decimal notation for fractions with denominators

463   10 or 100 (**4.NF.6▲**), understanding that the number of digits to the right of the

464   decimal point indicates the number of zeros in the denominator. Students make

465   connections between fractions with denominators of 10 and 100 and place value.

466   They read and write decimal fractions; for example, students say 0.32 as "thirty-

467   two hundredths" and learn to flexibly write this as both 0.32 and $\frac{32}{100}$.

468

Focus, Coherence, Rigor.

Teachers are urged to consistently use place value based language when naming decimals to reinforce student understanding, i.e., by saying "four tenths" when referring to 0.4, as opposed to "point four", and by saying "sixty eight hundredths" when referring to 0.68, as opposed to "point sixty eight" or "point six eight."

469

470   Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. Students reason

471   that $\frac{32}{100}$ is a little more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$ (or $\frac{4}{10}$). It is closer to $\frac{30}{100}$,

472   so it would be placed on the number line near that value. **(MP.2, MP.4, MP.5,**

473   **MP.7)**

0.32

474
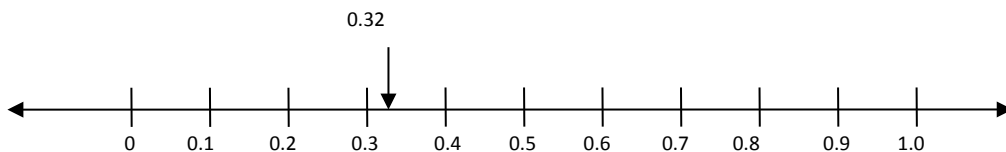
475   Students compare two decimals to hundredths by reasoning about their size

476   (**4.NF.7▲**).They relate their understanding of the place value system for whole

477   numbers to fractional parts represented as decimals. Students compare decimals

478   using the meaning of a decimal as a fraction, making sure to compare fractions

479   with the same denominator and that the "wholes" are the same.

480

481

Common misconceptions:

- Students sometimes treat decimals as whole numbers when making comparisons of two decimals, ignoring place value. For example, they think that $0.2 < 0.07$ simply because $2 < 7$.

- Students sometimes think the longer the decimal number the greater the value. For example they think that 0.03 is greater than 0.3.

482

483   **Domain: Measurement and Data**

484

| Measurement and Data |
| 4.MD |

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

485

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486   Students will need ample opportunities to become familiar with new units of

487   measure. In prior years, work with units was limited to units such as pounds,

488   ounces, grams, kilograms, and liters, and students did not convert

489   measurements. Students may use a two-column chart to convert from larger to

490   smaller units and record equivalent measurements. For example:

491

kg	g		ft	in		lb	oz
1	1000		1	12		1	16
2	2000		2	24		2	32
3	3000		3	36		3	48

492

493   Students in grade four begin using the four operations to solve word problems

494   involving measurement quantities such as liquid volume, mass, and time

495   (**4.MD.2**), including problems involving simple fractions or decimals.

496

497

Examples: Word Problems Involving Measures (4.MD.2).
1. **Division/fractions**: Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get? Students may record their solutions using fractions or inches.   *Solution:* The answer would be $\frac{2}{3}$ of a foot or 8 inches. Students are able to express the answer in inches because they understand that $\frac{1}{3}$ of a foot is 4 inches and $\frac{2}{3}$ of a foot is 2 groups of $\frac{1}{3}$.
2. **Addition:** Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?   *Solution:* Students know that every 60 minutes make an hour. We know she ran one hour which is 60 minutes. She also ran 15 + 25 +40 = 80 minutes more, which makes 140 total minutes.
3. **Multiplication:** Mario and his two brothers are selling lemonade. Mario brought one and a half liters, Javier brought 2 liters, and Ernesto brought 450 milliliters. How many total milliliters of lemonade did the boys have?   *Solution:* Students know that 1 liter is 1000 ml, so that Mario bought 1000 + 500 = 1500 ml, and Javier bought 2 × 1000 = 2000 ml. This means altogether they had 1500 + 2000 + 450 = 3950 ml.

498   (Adapted from Arizona 2012)

499

> **Focus, Coherence, Rigor:**
>
> In grade four students use the four operations to solve word problems involving measurement quantities such as liquid volume, mass and time (**4.MD.1-2**). Measurement provides a context for solving problems using the four operations and connects to and supports major work at the grade in the cluster "Use the four operations with whole number to solve problems" (**4.OA.1-3▲**) and clusters in the domain "Number and operations—Fractions" (**4.NF.1-4▲**). For example, students use whole-number multiplication to express measurements given in a larger unit in terms of a smaller unit and students solve word problems involving addition and subtraction of fractions or multiplication of a fraction by a whole number (Adapted from PARCC 2012).

500

Measurement and Data
4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

501

502 Students developed an understanding of area and perimeter in third grade by

503 using visual models. In grade four students are expected to use formulas to

504 calculate area and perimeter of rectangles; however, they still need to

505 understand and be able to communicate their understanding of why the formulas

506 work. It is still important for students to make length units or square units inside a

507 small rectangle to keep the distinction fresh and visual, and some students may

508 still need to write the lengths of all four sides before finding the perimeter.

509 Students know that answers for the area formula ($\ell \times w$) will be in square units,

510 and that answers for the perimeter formula ($2\ell + 2w$, or $2[\ell + w]$) will be in linear

511 units (Adapted from Arizona 2012).

512

> **Example: Area and Perimeter of Rectangles. (MP.2, MP.4)**
>
> Sally wants to build a pen for her dog Callie. Her parents give her $200 to buy the fencing material, but they want Sally to design the pen. Her parents suggest that she consider different plans. Her parents also remind her that Callie needs as much room as possible to run and play,

and that the pen can be placed anywhere in the yard and the wall of the house could be used as one side of the pen. Sally decides to buy fencing material that costs $8.50 per foot. She will also need at least one three foot wide gate for the pen that costs $15.

- Design a pen for Callie. Experiment with different pen designs and consider the advice from Sally's parents. Sally's house can also be any configuration.

Write a letter to Sally with your various diagrams and calculations. Explain why certain designs are better for Callie (Adapted from CMC Margaret DeArmond).

513

Measurement and Data
4.MD

Represent and interpret data.
4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

514

515 As students work with data in kindergarten through grade five, they build

516 foundations for the study of statistics and probability in grades six and beyond,

517 and they strengthen and apply what they are learning in arithmetic.

518

519 In grade four students make a line plot to display a data set of measurements in

520 fractions of a unit $\left(\frac{1}{2},\frac{1}{4},\frac{1}{8}\right)$ and they solve problems involving addition and

521 subtraction of fractions by using information presented in line plots (**4.MD.4**).

522

Example: Interpreting Line Plots.

Ten students measure objects in their desk to the nearest $\left(\frac{1}{2},\frac{1}{4},\frac{1}{8}\right)$ inch. They record their results on the line plot below (in inches).
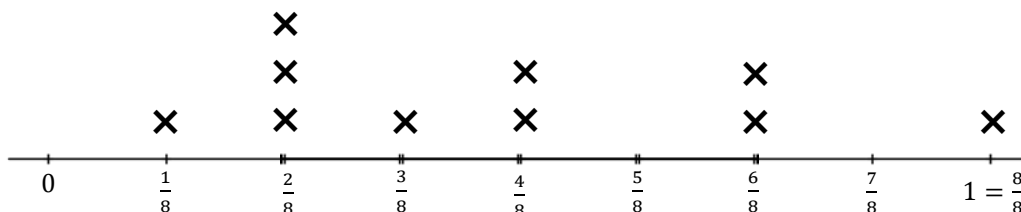
Possible related questions:

> - How many objects measured $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ inch?
>
> - If you put the objects end to end, what would the total length be?
>
> - If five $\frac{1}{8}$-inch pencils are placed end to end, what would be the total length?

523   (Adapted from Arizona 2012)

524

Measurement and Data
4.MD

Geometric measurement: understand concepts of angle and measure angles.
 5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
      a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
      b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
 6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
 7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

525

526   Students in grade four learn that angles are geometric shapes formed by two

527   rays that share a common endpoint. They understand angle measure as being

528   that portion of a circular arc that is formed by the angle when a circle is centered

529   at their shared vertex. The diagram helps students see that an angle is

530   determined by the arc it creates relative to the size of the entire circle, evidenced

531   by the picture showing two angles of the same measure though their circles are

532   not the same. However, the pie-shaped pieces formed by each angle are

533   different-sized; this shows that angle measure is not defined in terms of these

534   areas.
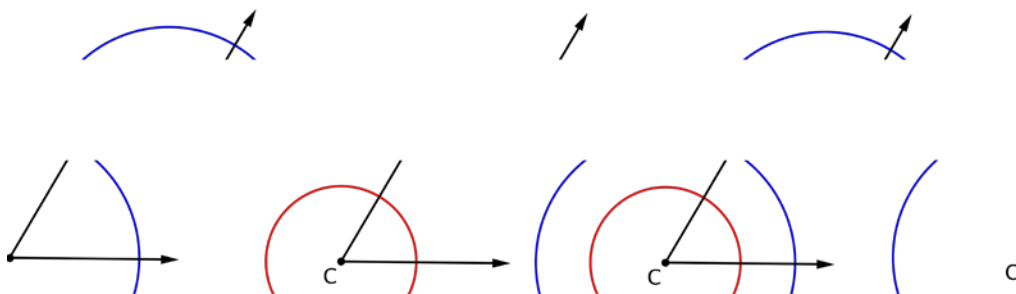
535

536   The angle in each case is 60°, since it measures an arc that is $\frac{1}{6}$ the total

537   circumference of the circle in both the blue and red circles. However, the pie-

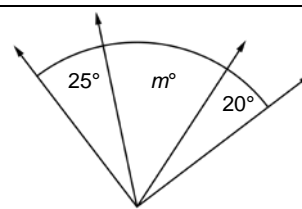538   slices that the angle forms have different areas.

539

540   Before students begin measuring angles with protractors **(4.MD.6)**, they need to

541   have some experience with benchmark angles. They transfer their understanding

542   that a 360⁰ rotation about a point makes a complete circle to recognize and

543   sketch angles that measure approximately 90⁰ and 180⁰. They extend this

544   understanding and recognize and sketch angles that measure approximately 45⁰

545   and 30⁰. Students use appropriate terminology (acute, right, and obtuse) to

546   describe angles and rays (perpendicular). Students recognize angle measure as

547   additive and use this to solve addition and subtractions problems to find unknown

548   angles on a diagram.

549

Examples: Angle Measure is Additive.
1. If the two rays are perpendicular (see 4.G.1), what is the value of m?   *Solution:* "Since perpendicular lines make an angle that measures 90°, I know that $25 + m + 20 = 90$, this means that $m = 90 - 45 = 45$."
2. Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30°. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?   *Solution:* "This looks like it is four times as much, so it is 4 × 30° = 120°."

(Adapted from Arizona 2012)

550

Focus, Coherence, and Rigor: Students' work with concepts of angle measures (**4.MD.5a and 7**) also connects to and supports adding fractions, which is major work at the grade in the cluster "Building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers" (**4.NF.3-4▲**). For example, a one degree measure is a fraction of an entire rotation and adding angle measures together is the same as adding fractions with a denominator of 360.

551

552 Before students solve word problems involving unknown angle measures

553 (**4.MD.7**), they need to understand concepts of angle measure (**4.MD.5**) and,

554 presumably, gain some experience measuring angles (**4.MD.6**). Students also

555 need some familiarity with the geometric terms that are used to define angles as

556 geometric shapes (**4.G.1**) (Adapted from PARCC 2012).

557

558   **Domain: Geometry**

559 A critical area of instruction in grade four is for students to understand that

560 geometric figures can be analyzed and classified based on their properties, such

561 as having parallel sides, perpendicular sides, particular angle measures, and

562 symmetry.

563

Geometry   4.G
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.   1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.   2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **(Two dimensional shapes should include special triangles, e.g., equilateral, isosceles, scalene, and special quadrilaterals, e.g., rhombus, square, rectangle, parallelogram, trapezoid.) CA**   3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

564

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565 Grade four is the first time students are exposed to rays, angles, and

566 perpendicular and parallel lines **(4.G.1)**. In addition, students

567 classify figures based on the presence and absence of parallel

568 or perpendicular lines and angles (**4.G.2**). It is helpful if

569 examples of points, line segments, lines, angles, parallelism,

570 and perpendicularity can be seen daily. For example a wall

571 chart with the images shown could be displayed in the

572 classroom.

573
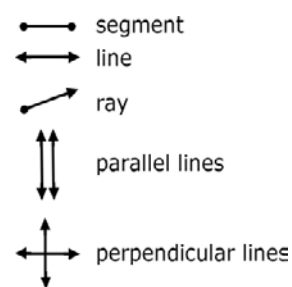
574 Students need to see all of these in different orientations. Students could draw

575 these in different orientations and decide if all of the drawings are correct. Also

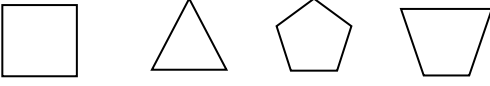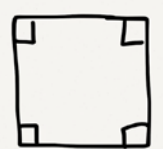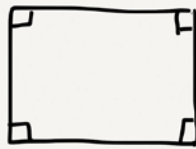576 they need to see the range of angles that are acute and obtuse.

577

578 Two-dimensional figures may be classified using different characteristics, such

579 as the presence of parallel or perpendicular lines or by angle measurement.

580 Students may use transparencies with lines drawn on them to arrange two lines

581 in different ways to determine that the two lines might intersect in one point or

582 may never intersect, thereby understanding the notion of parallel lines. Further

583 investigations may be initiated using geometry software. These types of

584 explorations may lead to a discussion on angles.

585

586 Students' prior experience with drawing and identifying right, acute, and obtuse

587 angles helps them classify two-dimensional figures based on specified angle

588 measurements. They use the benchmark angles of 90°, 180°, and 360° to

589 approximate the measurement of angles. Right triangles (triangles with one right

590 angle) can be a category for classification, with subcategories, e.g., an isosceles

591 right triangle has two or more congruent sides and a scalene right triangle has no

592 congruent sides.

593

Examples: Classifying Shapes According to Attributes. (MP.3)
1. Identify which of these shapes have perpendicular or parallel sides and justify your selection.

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2. Explain why a square is considered a rectangle but a rectangle isn't necessarily a square.

Solution: "I know that rectangles are four-sided shapes that have four right angles. This makes any square a rectangle since a square has four sides and four right angles also. *But*, a square is a *special* kind of rectangle. What I mean is that you can have a rectangle that has its sides not all equal, and then it isn't a square. I drew examples to show what I mean."

594

595 Finally, students recognize a line of symmetry for a two-dimensional figure as a

596 line across the figure such that the figure can be folded along the line into

597 matching parts.

598 (Adapted from Arizona 2012)

599

600 **Essential Learning for the Next Grade**

601

602 In kindergarten through grade five, the focus is on the addition, subtraction,

603 multiplication, and division of whole numbers, fractions, and decimals, with a

604 balance of concepts, procedural skills, and problem solving. Arithmetic is viewed

605 as an important set of skills and also as a thinking subject that, done thoughtfully,

606 prepares students for algebra. Measurement and geometry develop alongside

607 number and operations and are tied specifically to arithmetic along the way.

608 Multiplication and division of whole numbers and fractions are an instructional

609 focus in grades three through five.

610

611 To be prepared for grade five mathematics, students should be able to

612 demonstrate they have acquired certain mathematical concepts and procedural

613 skills by the end of grade four and have met the fluency expectations for the

614 grade. For fourth graders, the expected fluencies are to add and subtract multi-

615 digit whole numbers using the standard algorithm within 1,000,000 (**4.NBT.4▲**).

616   These fluencies and the conceptual understandings that support them are

617   foundational for work in later grades.

618

619   Of particular importance at grade four are concepts, skills, and understandings

620   needed to use the four operations with whole numbers to solve problems

621   (**4.OA.1-3▲**); generalize place value understanding for multi-digit whole numbers

622   (**4.NBT.1-3▲**); use place value understanding and properties of operations to

623   perform multi-digit arithmetic (**4.NBT.4-6▲**); extend understanding of fraction

624   equivalence and ordering (**4.NF.1-2▲**); build fractions from unit fractions by

625   applying and extending previous understandings of operations on whole numbers

626   (**4.NF.3-4▲**); and understand decimal notation for fractions, and compare

627   decimal fractions (**4.NF.5-7▲**).

628

629   Fractions

630   Fraction equivalence is an important theme within the standards. Understanding

631   fraction equivalence is necessary to extend arithmetic from whole numbers to

632   fractions and decimals. Students need to understand fraction equivalence and

633   that $\frac{a}{b} = \frac{n \times a}{n \times b}$. They should be able to represent equivalent common fractions and

634   apply this understanding to compare fractions and express their relationships

635   using the symbols, >, <, or =. Students understand how to represent and read

636   proper fractions, improper fractions, and mixed numbers in multiple ways.

637

638   Grade four students should understand addition and subtraction with fractions

639   having like denominators. This understanding represents a multi-grade

640   progression as students add and subtract fractions here in grade four with like

641   denominators by thinking of adding or subtracting so many unit fractions.

642   Students should be able to solve word problems involving addition and

643   subtraction of fractions referring to the same whole and having like

644   denominators, e.g., by using visual fraction models and equations to represent

645   the problem. Students should understand how to add and subtract proper

646   fractions, improper fractions, and mixed numbers with like denominators.

647

648 Students extend their developing understanding of multiplication to multiply a

649 fraction by a whole number. To support their understanding students should

650 understand a fraction as the numerator times the unit fraction with the same

651 denominator. Students should be able to rewrite fractions as multiples of the unit

652 fraction of the same denominator, multiply a fraction by a whole number using a

653 visual model, and use equations to represent problems involving the

654 multiplication of a fraction by a whole number by multiplying the whole number

655 times the numerator.

656

657 Four operations with whole numbers

658 By the end of grade four, students should fluently add and subtract multi-digit

659 whole numbers to 1,000,000 using the standard algorithm. Students should also

660 be able to use the four operations to solve multi-step word problems with whole

661 number remainders.

662

663 In grade four students develop their understanding and skills with multiplication

664 and division. Students combine their understanding of the meanings and

665 properties of multiplication and division with their understanding of base-ten units

666 to begin to multiply and divide multi-digit numbers. Grade four students should

667 know how to express the product of two multi-digit numbers as another multi-digit

668 number. They also should know how to find whole-number quotients and

669 remainders with up to four-digit dividends and one-digit divisors. Representing

670 multiplication and division using a rectangular area model helps students

671 visualize these operations. This work will develop further in grade five and

672 culminate in fluency with the standard algorithms in grade six.

673

674   ## Grade 4 Overview
675

676   ### Operations and Algebraic Thinking

677   • Use the four operations with whole numbers to solve
678   problems.

679   • Gain familiarity with factors and multiples.

680   • Generate and analyze patterns.

681
682   ### Number and Operations in Base Ten

683   • Generalize place value understanding for multi-digit whole
684   numbers.

685   • Use place value understanding and properties of operations to
686   perform multi-digit arithmetic.

687
688   ### Number and Operations—Fractions

689   • Extend understanding of fraction equivalence and ordering.

690   • Build fractions from unit fractions by applying and extending
691   previous understandings of operations on whole numbers.

692   • Understand decimal notation for fractions, and compare decimal fractions.

693
694   ### Measurement and Data

695   • Solve problems involving measurement and conversion of measurements from a
696   larger unit to a smaller unit.

697   • Represent and interpret data.

698   • Geometric measurement: understand concepts of angle and measure angles.

699
700   ### Geometry

701   • Draw and identify lines and angles, and classify shapes by properties of their lines
702   and angles.

Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

703   ## Grade 4

704

| **Operations and Algebraic Thinking** | **4.OA** |

Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]

3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

| **Number and Operations in Base Ten**[2] | **4.NBT** |

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.*

2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

3. Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.

5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

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706
707   [1]See Glossary, Table 2.
708   [2]Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
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Number and Operations—Fractions[3]	**4.NF**

Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

    a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

    b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

    c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

    d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

    a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

    b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

    c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using **the number line or another** visual model. **CA**

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716 [3]Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
717 [4]Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in
718 general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.
719

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| **Measurement and Data** | **4.MD** |

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Represent and interpret data.

4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

    a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

    b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

| **Geometry** | **4.G** |

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **(Two dimensional shapes should include special triangles, e.g., equilateral, isosceles, scalene, and special quadrilaterals, e.g., rhombus, square, rectangle, parallelogram, trapezoid.) CA**

3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

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